1. First and second derivatives.

   a. f(x)=$\frac{5\sin(x)}{3} - x$

      $\frac{d}{dx}$(cos(x)) = -sin(x)

      $\frac{d}{dx}$(sin(x)) = cos(x)

      f'(x) = $\frac{5\cos(x)}{3} - 1$

      f''(x) = $\frac{-5\sin(x)}{3}$

   b. y =$\frac{x}{2}$ - $\frac{\sin(2x)}{4}$

      $\frac{d}{dx}$(sin(2x)) = 2cos(2x) …. differentiating using chain rule where u =2x

      y'(x) = $\frac{1}{2}$ - $\frac{\cos(2x)}{2}$

      $\frac{d}{dx}$(cos(2x)) = -2sin(2x) …. differentiating using chain rule where u =2x

      y''(x) = sin(2x)

2. Find the derivative.

   a. y = sec^2(πx)

   sec^2(πx) = (sec(πx))^2

   $\frac{d}{dx}$(sec(πx)) = sec(πx)tan(πx)* $\frac{d}{dx}$(πx)

   = πsec(πx) tan(πx)

   Applying Chain Rule: u = sec(πx)

   $\frac{d}{dx}$ (sec^2(πx)) = 2(u)(u')

   = 2(sec(πx)) (πsec(πx) tan(πx))

   = 2πsec^2(πx) tan(πx))

b. y = $sin^2(\frac{x}{2})$

$sin^2(\frac{x}{2})$ = $\left(\sin\left(\frac{x}{2}\right)\right)^2$

Applying Chain Rule: u = $\sin\left(\frac{x}{2}\right)$

$$\frac{du}{dx} = \frac{1}{2}\cos\left(\frac{x}{2}\right)$$

$\frac{d}{dx}(sin^2(\frac{x}{2}))$ = 2(u)*u'

= $\sin\left(\frac{x}{2}\right)$($\cos\left(\frac{x}{2}\right)$)

= $\frac{\sin(x)}{2}$

3. f(x) = $\frac{2}{\sqrt{x+1}}$

a. Find f'(x)

$$\frac{2}{\sqrt{x+1}} = 2(x+1)^{-\frac{1}{2}}$$

Applying Chain Rule: u = x+1

$\frac{d}{dx}(2(x+1)^{-\frac{1}{2}})$ = -1$(x+1)^{-\frac{3}{2}}$

= $\frac{-1}{(x+1)^{\frac{3}{2}}}$

b. $\frac{-1}{(x+1)^{\frac{3}{2}}} = -\frac{1}{8}$

$(x+1)^{\frac{3}{2}} = 8$
$(x+1)^3$ = 64
$x + 1 = 4$

x = 3

4. f(x) = $\frac{x^3}{3}$ - $\frac{3x^2}{2}$ + 1

a. Find the derivative

F'(x) = x^2-3x

b. 8x -2y = 1
y = 4x - $\frac{1}{2}$
m = 4

4 = x^2-3x

Solving the quadratic equation, x = 4, x = -1

f(4)= $\frac{4^3}{3}$ - $\frac{3(4)^2}{2}$ + 1 = $\frac{-5}{3}$

f(-1) = $\frac{(-1)^3}{3}$ - $\frac{(-1)^2}{2}$ + 1 = $\frac{-5}{6}$

= (4, $\frac{-5}{3}$) and (-1, $\frac{-5}{6}$)

5. f(x) = $\frac{27}{x^2+2}$

a. Find its derivative.

Applying Quotient Rule:

$\frac{d}{dx}(\frac{27}{x^2+2})$ = $\frac{-(2x)*27}{(x^2+2)^2}$

= $\frac{-54x}{(x^2+2)^2}$

b. f(1) = 9

m = f'(1) = $\frac{-54}{(1+2)^2}$ = -6

m – (9) = -6(x - 1)

y = -6x + 15

6. f(x) = 1 – cos(2x) + cos^2(x)

   a. Find the derivative.

      f'(x) = (-2sin(2x)) -2sin(x)cos(x)

      = 2sin(2x) – sin(2x)
      = sin(2x)

   b. sin(2x) = 0

      2x = 0; x = 0
      2x = π; x = $\frac{\pi}{2}$
      2x = 2 π; x = π
      2x = 3 π; x = $\frac{3\pi}{2}$
      2x = 4 π; x = π

      x = 0, x = $\frac{\pi}{2}$, x = π, x = $\frac{3\pi}{2}$, x = π

7. $6x^2y - \pi$cos(y) = 7π

   a. Find $\frac{dy}{dx}$.

      $\frac{d}{dx}(6x^2y)$ - $\frac{d}{dx}(\pi\cos(\text{y}))$ = $\frac{d}{dx}(7\pi)$

      12xy + $6x^2\frac{dy}{dx}$ + $\pi\sin(\text{y})$ $\frac{dy}{dx}$ = 0

      $6x^2\frac{dy}{dx}$ + $\pi\sin(\text{y})\frac{dy}{dx} = -12xy$

      $$\frac{dy}{dx} = \frac{-12xy}{6x^2 + \pi\sin(\text{y})}$$

   b. $M_{\text{tangent line}} = \frac{-12\pi}{6+\pi\sin(\pi)} = -2\pi$

      $M_{\text{normal line}} = \frac{1}{2}\pi$

      y – (π) = $\frac{1}{2}$π(x - 1)

      y = $\frac{1}{2}$π(x+1)

8. a. $x^2 + y^2 = c^2$

$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(c^2)$

$2x\frac{dx}{dx} + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$, but $\frac{dc}{dt} = 0$

$(8)(-5) + 15(\frac{dy}{dt}) = 0$

$\frac{dy}{dt} = \frac{8}{3}$ ft/sec

b. $\frac{y}{x} = \tan\theta$

$y = x \tan\theta$

$\frac{dy}{dt} = \tan\theta\frac{dx}{dt} + x\, sec^2\theta\frac{d\theta}{dt}$

$-5 = (\frac{8}{15})(\frac{8}{3}) + (15)(\frac{17}{15})^2\ \frac{d\theta}{dt}$

$\frac{d\theta}{dt} = \frac{-1}{3}$rad/s

9. $S = 6a^2$

$\frac{dS}{dt} = 12a\frac{da}{dt}$

$$\frac{dS}{dt} = 72in^2/sec$$

$72 = 12a\frac{da}{dt}$; a = 3

$\frac{da}{dt} = 2in$/sec

$V = a^3$

$\frac{dV}{dt} = 3a^2 * \frac{da}{dt}$

$$\frac{dV}{dt} = 3 * 9 * 2$$

$=54\ in^3$/sec

10. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(\frac{h}{2})^2 h$

$V = \frac{1}{12}\pi h^3$

$$\frac{dV}{dt} = \frac{dV}{dh} * \frac{dh}{dt}$$

$2 = \frac{d}{dh}(\frac{1}{12}\pi h^3)\,\frac{dh}{dt}$

$8 = \pi h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{8}{\pi h^2}$ where h = 3

$\frac{dh}{dt} = \frac{8}{9\pi}$ m/min